I-1. THEORY OF A THERMAL-GRADIENT GAS LENS

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A communications system using a carrier at light frequencies requires a guiding medium to assure light propagation from transmitter to receiver. A periodic sequence of lenses can act as a waveguide if the lenses are properly spaced (Reference 1). Lenses made of a solid dielectric such as glass not only absorb some of the light in the dielectric medium itself but also inevitably scatter some of the light out of the beam at the interface between air and dielectric. Since many closely spaced lenses are required to guide the beam around intentional and accidental bends (Reference 2) of the transmission path the loss of a lens-waveguide can become quite high. These losses can be avoided if instead of solid dielectrics gasses are used as lens material.

Gas lenses have recently been described in the literature (References 3 and 4). The present paper presents the theory of the ray optics of a gas lens. Figure 1 shows the geometry of the gas lens. The lens is formed by blowing a cool gas into a round tube with a higher but constant wall temperature. As the gas heats up in the tube a temperature gradient is established since the gas becomes warm close to the wall and remains cool in the center of the tube.

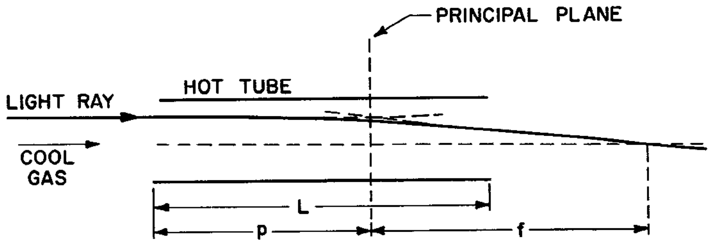

Figure 1. Geometry of Gas Lens

The theory of the temperature distribution is well known (Reference 5) provided that one neglects the temperature dependence of the heat conductivity and density of the gas. The known temperature distribution T can then be used to calculate the density distribution throughout the lens. Gas density ρ and index of refraction n are related by

$$n - 1 = (n_o - 1)\frac{\rho}{\rho_o} = (n_o - 1)\frac{T_o}{T} . \quad (1)$$

The index o refers to the properties of the cool gas at the input.

The ray optics of the lens is described by the paraxial ray equation (Reference 6),

$$\frac{d^2r}{dz^2} = \frac{1}{n}\frac{dn}{dr}, \tag{2}$$

with r distance of the ray from the tube axis and z distance measured along the axis.

The ray Equation (2) cannot be solved exactly due to the complicated form of the function describing the temperature distribution. An approximate description was used which treats the solution of Equation (2) in two ranges. The gas enters the warm tube at a constant temperature. Close to the input the temperature distribution is given by an infinite series. Since the ray does not change position very rapidly in the lens a good approximation is obtained by taking $\frac{dn}{dr} = f(z)$ and neglecting the change of $\frac{dn}{dr}$ as a function of r.

However, the temperature distribution T(r,z) can be described by the first term of the above mentioned series a short distance from the gas input. This first term can be approximated by a function of the form

$$T_w - T(r,z) = (a + br^2 + cr^4)\, e^{-\gamma z}, \tag{3}$$

where

T_w = temperature of hot tube.

With this form of the temperature distribution Equation (2) can be solved for rays close to the axis ($r \ll a$, a = tube radius). For rays at arbitrary distance from the tube axis the solution can be given by a Taylor series expansion.

The ray trajectory can be used to compute the focal length and the principal plane of the gas lens.

Figure 2 shows the focal length (measured from the principal plane) as a function of gas velocity normalized with respect to the tube length L for rays close to the tube axis. The gas velocity is normalized with respect to a velocity V given by

$$V = \frac{kL}{a^2 \rho c_p}, \tag{4}$$

where

k = heat conductivity

c_p = specific heat at constant pressure.

The parameter AL labeling the curves of Figure 2 is given by

$$AL = \sqrt{5.35\,(n_o - 1)\,\frac{T_w - T_o}{T_o}\,\frac{L}{a}}, \tag{5}$$

where

T_o = temperature of cool gas at tube input.

Figure 2 shows that an optimum flow rate exists minimizing the focal length of the gas lens.

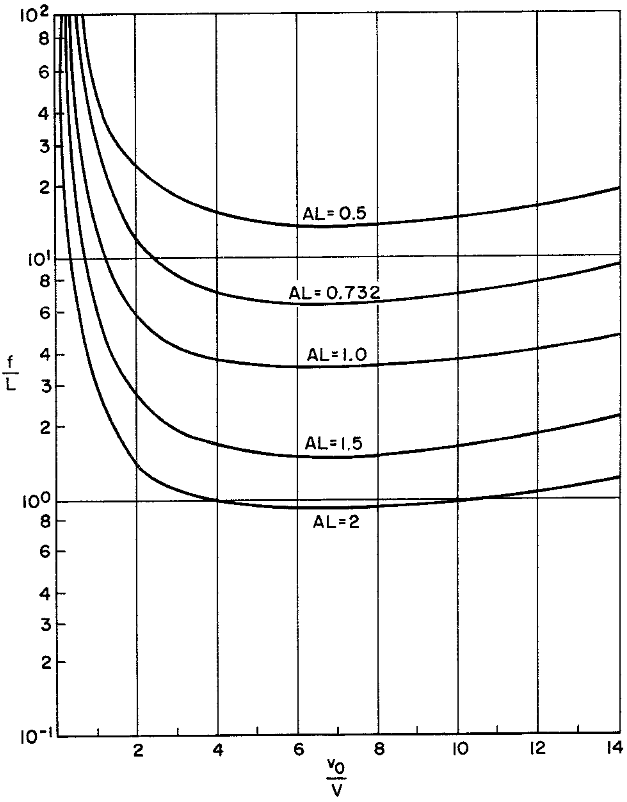

Figure 2. Focal Length Divided by Length of Lens as a Function of Normalized Gas Velocity

Figure 3 shows the position of the principal plane for rays close to the tube axis. (For definition of p see Figure 1.) The principal planes of rays moving in opposite directions coincide closely as long as $AL < 2$. This means that the gas lens behaves like a thin lens in spite of its appearance.

The focal length as a function of the rays input position r_o is plotted in Figure 4 for two values of the gas velocity. At $\frac{v_o}{V}$ = 10 the focal length depends on the position of the ray in the lens, the lens distorts For $\frac{v_o}{V} = 6.5$ there is hardly any dependence of focal length on ray position. It is a lucky coincidence that this occurs at the same flow velocity which minimizes the focal length. A plot of the position of the principal plane as a function of the ray's input position (Figure 5) reveals that the principal plane is not independent of the ray's position even though the focal length is The principal plane is somewhat warped.

It can be said in conclusion that the gas lens can be described by a nearly distortion-free thin lens which is warped to fit the principal plane of Figure 5 if operated at values $\frac{v_o}{V} = 6.5$.

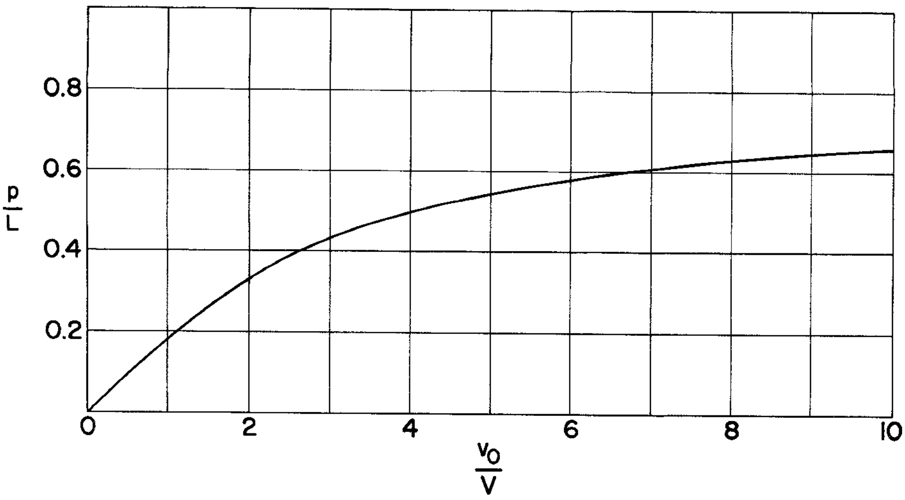

Figure 3. Position of Principal Plane as a Function of Normalized Gas Velocity

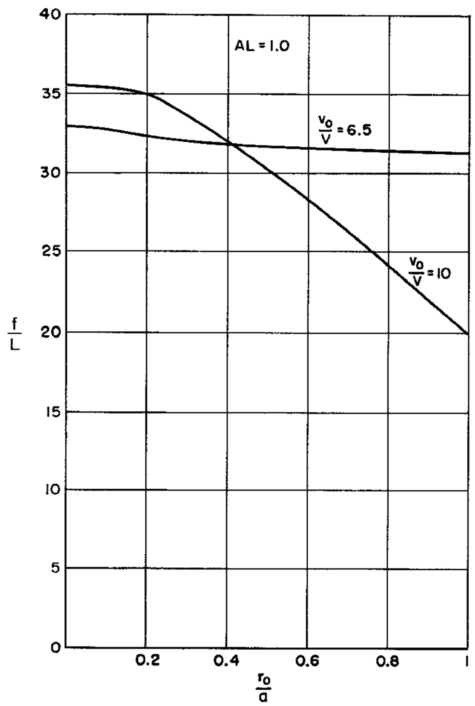

Figure 4. Focal Length Divided by Tube Length as a Function of the Ray's Input Position

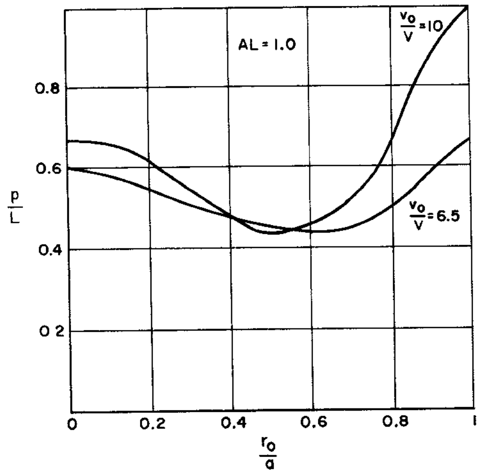

Figure 5. Position of Principal Plane as a Function of the Ray's Input Position

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